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LETTER TO THE EDITOR

Double minimum in the free energy of a type-II superconductor

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Abstract. We find that the free energy of a type-II superconductor as a function of magnetic field displays two minima and one maximum value. Thus there is a new type of oscillatory state tunnelling from one minimum to the other. The origin of this state occurs in the proper consideration of the flux-lattice energy for a two-dimensional system with flux quantization.

In the original Abrikosov solution [1], the free energy of a type-II superconductor as a function of the magnetic field in two dimensions describes periodic patterns due to flux quantization of the magnetic induction. In these patterns the lines of forces do not cross each other. The symmetry of the patterns changes from the triangular lattice at low fields to the square lattice at high fields with suitable emission of Goldstone bosons which carry the missing symmetries. Abrikosov found that the flux lines begin to penetrate when $H\varphi_0/4\pi$ becomes comparable with the energy per unit length of a single flux line. Here H is the magnetic field and $\varphi_0 = hc/2e$. In fact, this is the only definition of H_{c1}. There is a phase in which the flux lines are entangled with each other and there is another with well separated flux lines as described by Nelson [2]. One of us has found [3] that the fluxons should be treated as bosons for a proper theory of flux-lattice melting.

In this letter we calculate the flux-lattice energy which in two dimensions contains the quantized area in terms of the inverse magnetic induction and the unit flux. The free energy then has a term in inverse square of magnetic induction so that upon minimization as a function of magnetic induction, two minima are found. Thus we predict a new kind of intermediate flux state in type-II superconductors. This new phase should be accessible in high-temperature superconductors.

The commensurate–incommensurate transition in two dimensions has been studied by Coppersmith *et al* [4]. In this problem, fluctuations contribute to the wall free energy and cause an effective repulsive interaction between walls that varies as $1/l^3$ where l is the average distance between the walls. This repulsive interaction between the walls makes the comensurate–incommensurate transition to the striped phase continuous. The lattice constant is a. The number of possible positions for a given hexagon is of the order of $(l/a)^\delta$ yielding an entropic contribution to the free energy per hexagon of the form $-\delta T \ln|l/a|$ where $\delta \simeq 1$. We define a dimensionless reciprocal lattice vector as $q = l/a$. This entropic contribution is useful for writing the free energy.

The collisions between the flux lines make a contribution of $k_B \ln q$ to the entropy of the system with $q > 1$. Since the spacing between collisions in the z-direction is,

$l \simeq \varepsilon_1/k_B T n$, the total number of collisions is of the order of $(L/l)\ N = LA\ n^2 k_B T/\varepsilon_1$ where A is the area of the cross section and ε_1 is the energy per unit length of a single flux line. The vortices form a triangular lattice with areal density, $n = B/\varphi_0$, and N is the number of flux lines in a sample of length L. The statistically averaged Gibbs free energy per unit volume, $g(n)$, when minimized with respect to n, gives the magnetic induction

$$B = (2\varphi_0/\sqrt{3}\lambda^2)\{\ln[3\varphi_0/4\pi\lambda^2(H - H_{c1})]\}^{-2} \tag{1}$$

for which $g(n)$ is minimum. Here the logarithmic term has been included by analogy with the problem of Bose condensation, and λ is the London penetration depth. This is the magnetic induction of a type-II superconductor first found by Abrikosov and written here in the form suggested by Nelson [2].

We write the free energy as

$$g(n) = g(0) + (\varepsilon_1 - H\varphi_0/4\pi)n + n(3\varphi_0^2/8\pi^2\lambda^2)K_0(d/\lambda) + (k_B T)^2 n^2 \ln q/\varepsilon_1 \tag{2}$$

where K_0 is the modified Bessel function and $d \sim n^{-1/2}$ is the line spacing. We minimize $g(n)$ by setting $\partial g(n)/\partial n = 0$, so that

$$\varepsilon_1 - H\varphi_0/4\pi + [3\varphi_0^2/(8\pi^2\lambda^2)]K_0(d/\lambda) + 2n(k_B T)^2 \ln q/\varepsilon_1 = 0. \tag{3}$$

We substitute the flux-quantization condition $n = B/\varphi_0$ in (3) to find,

$$B = [\varepsilon_1\varphi_0/2(k_B T)^2 \ln q][H\varphi_0/4\pi - 3\varphi_0^2 K_0(d/\lambda)/8\pi^2\lambda^2 - \varepsilon_1]. \tag{4}$$

Keeping only the first term, replacing $8\pi \ln q$ by $1/c$, a geometrical constant and replacing H by $H - H_{c1}$, we find that the dominant term in the magnetic induction is given by

$$B \simeq c\varepsilon_1\varphi_0^2(H - H_{c1})/(k_B T)^2. \tag{5}$$

In analogy with the problem of Bose condensation, we multiply $H - H_{c1}$ by $\ln(n/c)$ so that the magnetic induction is determined by

$$B \simeq c\varepsilon_1\varphi_0^2(H - H_{c1}) \ln(n/c)/(k_B T)^2. \tag{6}$$

This expression is obtained by ignoring the second term of (4) which describes the decay of magnetic induction as a function of distance. The expression (4) is therefore a better description of magnetic induction than (6).

In the case of two dimensions, we calculate the lattice energy. Since the area is flux quantized it depends on the magnetic field. Therefore, we consider the flux-lattice energy in d-dimensions as,

$$E_d = I_d L^d \hbar (k_B T)^{d+1}[(2\pi)^{d-1} v^d \hbar^{d+1}]^{-1} \tag{7}$$

where

$$I_d = \int x^d \, dx [e^x - 1]^{-1} \tag{8}$$

and v is the fluxon velocity. For $d = 2$, the above energy becomes

$$E_2 = (L^2 \hbar I_2 / 2\pi v^2)(k_B T/\hbar)^3. \tag{9}$$

Due to flux-quantized area, $L^2 = 1/n$, in the type-II superconductors so that $BL^2 = \varphi_0$ for one unit flux. Eliminating L^2 from (9) we find

$$E_2 = (\hbar I_2 / 2\pi n v^2)(k_B T/\hbar)^3. \tag{10}$$

The Gibbs free energy then becomes

$$g(n) = g(0) + (\varepsilon_1 - H\varphi_0/4\pi)n + n(3\varphi_0^2/8\pi^2\lambda^2)K_0(d/\lambda) + [(k_B T)^2 n^2/8\pi c\varepsilon_1] + (\hbar I_2/2\pi n v^2)(k_B T/\hbar)^3. \tag{11}$$

We evaluate $\partial g(n)/\partial n = 0$ and set $n = B/\varphi_0$ to find

$$[(k_B T)^2/4\pi c\varphi_0\varepsilon_1]\, B^3 - B^2[H\varphi_0/4\pi - \varepsilon_1 - (3\phi_0^2/8\pi^2\lambda^2)\, K_0(d/\lambda)] - (\hbar I_2 \varphi_0^2/2\pi v^2)(k_B T/\hbar)^3 = 0 \tag{12}$$

which is cubic in B, the roots of which occur at the maxima and minima in the free energy $g(n)$. Ignoring the small terms in the coefficient of B^2 we find that (12) may be written as

$$H - H_{c1} = (k_B T)^2 B / c\varepsilon_1\varphi_0^2 - 2I_2\varphi_0(k_B T)^3/v^2\hbar^2 B^2 \tag{13}$$

for which there are maxima and minima in $g(n)$. Ignoring the first term we see that $B \sim T^{3/2}$ which has the correct universal exponent independent of the system. The exponent of 3/2 was also found by de Almeida and Thouless [5] for a ferromagnetic transition. Usually the exponents depend on the dimensionality and the symmetry index of the system. The second term of (13) is found here for $d = 2$. The energy (7) is quantum mechanically correct for a Bose system. Indeed, Achar [6] has shown that the quantum nature of the fluid is important.

We abbreviate the coefficients of (12) as,

$$A_0 = -2cI_2\varphi_0^3 k_B T\varepsilon_1(v^2\hbar^2)^{-1} \tag{14a}$$

$$A_2 = -cH\varphi_0^2\varepsilon_1/(k_B T)^2 \tag{14b}$$

and

$$-f_0 = A_0 + (2/27)A_2^3 \tag{14c}$$

$$3f_1 = A_2^2 \tag{14d}$$

when the condition

$$27f_0^2 < 4f_1^3 \tag{15}$$

is satisfied, there are three real roots:

$$B_1 = (A_2/3)[2\cos(\varphi/3 - 1)] \tag{16a}$$

$$B_2 = -(A_2/3)[2\cos[(\pi - \varphi)/3] + 1] \tag{16b}$$

$$B_3 = -(A_2/3)[2\cos[(\pi + \varphi)/3] + 1] \tag{16c}$$

with

$$\cos\varphi = \tfrac{1}{2}(3/f_1)^{3/2} f_0 \tag{17}$$

for the magnetic induction. This means that as we go through the extrema of $g(n)$, the magnetic induction changes sign. When condition (15) is not satisfied but $27f_0^2 > 4f_1^3$, there is one real root:

$$B_0 = (A_2/3)[2\cosh(\varphi/3) - 1] \tag{18}$$

where

$$\cosh\varphi = \tfrac{1}{2}(3/f_1)^{3/2} f_0 \tag{19}$$

and there are two complex roots indicating decay of the magnetic induction. The largest terms of (2) are

$$g(n) = (k_B T)^2 n^2 \ln q/\varepsilon_1 - (H\varphi_0/4\pi)n \tag{20}$$

so that

$$\begin{aligned} g_1 &\simeq (k_B T)^2 B_1^2 \ln q/\varphi_0^2\varepsilon_1 - H|B_1|/4\pi \\ g_2 &\simeq (k_B T)^2 B_2^2 \ln q/\varphi_0^2\varepsilon_1 + H|B_2|/4\pi \\ g_3 &\simeq (k_B T)^2 B_3^2 \ln q/\varphi_0^2\varepsilon_1 + H|b_3|/4\pi \end{aligned} \tag{21}$$

where the modulus of B_i ($i = 1$–3) has been taken. At some of the angles, B_1 is negative while B_2 and B_3 are positive. Thus there is a minimum at B_1 and there are two maxima: one at B_2 and the other at B_3. When the value of φ enters the third quadrant, B_1 is positive while B_2 and B_3 are negative. Thus, B_2 and B_3 are two stable minima so that the system oscillates between g_2 and g_3 by tunnelling. From (17) $\cos\varphi$ is negative and hence two minima in $g(n)$ are predicted.

In conclusion, we find that the free energy of a type-II superconductor oscillates from one minimum value to another, each of which occurs at different fields. The multiplicity in the induction fields corresponding to minima in the free energy is caused by the flux-lattice energy per unit area.

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